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LETTER TO THE EDITOR

Matter field infinities in axial gauge gravity

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Abstract. The one-loop infinities for scalar, spinor and vector fields due to gravitational interactions are computed in the axial gravity gauge for the Einstein–Hilbert action. The results reveal an *axis dependence*, in addition to their expected non-renormalisability, which is expected to get progressively worse in higher orders of perturbation theory. This means that gauge equivalences between covariant and non-covariant gauges are impossible in ordinary quantum gravity (unlike renormalisable theories such as flavourdynamics or chromodynamics) unless cancellation of infinities occurs through some supersymmetric mechanism or unless a renormalisable conformal version of gravity is adopted.

Studies of the one-loop gravitational infinities in pure Einstein gravity were recently undertaken (Delbourgo 1981, Capper and Leibbrandt 1981a, b) for the axial gravity gauge,

$$n^{\mu}(g_{\mu\nu}-\eta_{\mu\nu})=0. \tag{1}$$

They showed that, in addition to their expected non-renormalisable character, the counterterms carried an explicit axis (n-) dependence among other things. This is very different from QCD, say, where the counterterms in the axial vector gauge are renormalisable and axis independent. In this note we complete the investigation for quantum gravity and report on the infinities associated with other sources of the gravitational field, namely scalar, spinor and vector particles. Again we find a persistent axis dependence in our answers which is deeply tied to the non-renormalisable character of the usual Einstein–Hilbert action, and which effectively puts the last nails into the coffin of the axial gravitational gauge. Only if the offending counterterms can be made to cancel, as in supergravity, or if a renormalisable (Weyl) version of gravity is adopted, will it be worthwhile to resurrect the corpse; since then, presumably, the n dependence will again disappear (as in QCD) and the renormalisation procedure become viable once more; in those circumstances it ought to be possible to establish equivalences (Beven and Delbourgo 1980) between the axial gravity gauge and other Lorentz covariant gauge choices, as one can with QCD.

We shall only sketch out the more salient features associated with the calculations: the details are long and extremely tiresome, and they can be straightforwardly reproduced by any diligent worker. We take

$$\begin{aligned}\mathscr{L}=\mathscr{L}_{\mathrm{E}}&+(-g)^{1/2}(\tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\tfrac{1}{2}m^2\phi^2+\xi R\phi^2)\\&+(-g)^{w+1/2}[\tfrac{1}{2}\mathrm{i}e^{n\mu}\bar{\psi}\gamma_n\partial_\mu\psi-\bar{\psi}(m+\tfrac{1}{4}\mathrm{i}\varepsilon^{klmn}B_{klm}\gamma_n\gamma_5)\psi]\\&-\tfrac{1}{4}(-g)^{1/2}g^{\mu\kappa}g^{\nu\lambda}F_{\mu\nu}F_{\kappa\lambda}\end{aligned} \tag{2}$$

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where $\mathscr{L}_E$ is the Palatini Lagrangian, e is the vierbein field and B is the spin connection,

$$B_{lmn} = e_l^{\lambda} e_m^{\mu}[\tfrac{1}{2}e_n^{\nu}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\nu g_{\mu\lambda}) - \partial_\lambda e_{n\mu}]. \tag{3}$$

We have chosen canonical weights for the scalar and vector fields but have afforded the luxury (Isham *et al* 1971) of an arbitrary weight w for the spinor field; also we have included an improvement term in the scalar case, parametrised by ξ, the optimal conformal choice being $\xi = -\frac{1}{12}$.

The graviton field will be defined as the fluctuation about the flat metric, $g_{\mu\nu} = \eta_{\mu\nu} + Kh_{\mu\nu}$ and we work in the axial gravitational gauge (1). Hence the gravitational propagator is

$$\Delta_{\kappa\lambda,\mu\nu}(k) = [\mathscr{A}_{\kappa\mu}\mathscr{A}_{\lambda\nu} + \mathscr{A}_{\kappa\nu}\mathscr{A}_{\lambda\mu} + \mathscr{A}_{\kappa\lambda}\mathscr{A}_{\mu\nu}(1-l)^{-1}]/2k^2 \tag{4a}$$

in $2l$ dimensions, where

$$\mathscr{A}_{\mu\nu} = -\eta_{\mu\nu} + (k_\mu n_\nu + k_\nu n_\mu)/k \cdot n - k_\mu k_\nu n^2/(k \cdot n)^2. \tag{4b}$$

The gauge-compensating terms are completely irrelevant (Capper *et al* 1974) in what follows and will therefore be ignored. Also the spin connection does not contribute in one-loop order and can be disregarded at that level. From the Lagrangian (2) we can extract the graviton-scalar vertex function,

$$\Gamma_{\mu\nu}(p,q) = \tfrac{1}{2}K[p_\mu q_\nu + q_\mu p_\nu - \eta_{\mu\nu}(p \cdot q + m^2)] + 2\xi K[(p+q)_\mu (p+q)_\nu - (p+q)^2 \eta_{\mu\nu}], \tag{5}$$

the graviton-spinor vertex,

$$\Gamma_{\mu\nu}(p,q) = \tfrac{1}{8}K[(p-q)_\mu \gamma_\nu + (p-q)_\nu \gamma_\mu] - K\eta_{\mu\nu}(w + \tfrac{1}{2})[\tfrac{1}{2}(p-q)\cdot\gamma + m], \tag{6}$$

and the graviton-vector vertex,

$$\begin{aligned}\Gamma_{\mu\nu,\alpha\beta}(p,q) = \tfrac{1}{2}K[&(p \cdot q\eta_{\alpha\beta} - p_\beta q_\alpha)\eta_{\mu\nu} - (p_\mu q_\nu + p_\nu q_\mu)\eta_{\alpha\beta} - p \cdot q(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) \\ &+ p_\beta(q_\nu \eta_{\mu\alpha} + q_\mu \eta_{\nu\alpha}) + q_\alpha(p_\nu \eta_{\mu\beta} + p_\mu \eta_{\nu\beta})].\end{aligned} \tag{7}$$

Observe that (7) is photon gauge invariant,

$$p^\alpha \Gamma_{\mu\nu,\alpha\beta} = q^\beta \Gamma_{\mu\nu,\alpha\beta} = 0$$

in accordance with the structure of the stress tensor for the electromagnetic field,

$$F_{\mu\lambda}F_\nu^{\ \lambda} - \tfrac{1}{4}\eta_{\mu\nu}F_{\kappa\lambda}F^{\kappa\lambda}.$$

From (7) one may deduce that the gravitational correction to vacuum polarisation is photon gauge independent and therefore equals the Fermi gauge value.

The calculations amount to sewing together the vertices (6), (7) and (5) with the gravitational propagator (4) and that of the matter field. Tadpole contributions are, of course, dropped. Since we shall content ourselves with expressing the infinite parts, connected with counter Lagrangians, the simplest calculational procedure will be used (Beven and Delbourgo 1978) rather than the full machinery of dimensional regularisation. Our answers contain the logarithmic divergence factor $L = (16\pi^2)^{-1} \log \Lambda^2/\mu^2$, interpreted as the pole part $[16\pi^2(2-l)]^{-1}$ in dimensional regularisation, and are as follows†. The scalar self-energy infinity is

$$\Pi(p)|_\infty = K^2L\{(p^2-m^2)^2+(p^2-m^2)[10(p\cdot n)^2/3n^2+5p^2/12] + 8\xi(p\cdot n)^2(p^2-m^2)/n^2-4\xi m^2p^2-6\xi^2m^2(m^2+p^2)\}. \tag{8}$$

The spinor infinity is

$$\begin{aligned}\Sigma(p)|_\infty = \frac{K^2L}{32}[\![&(\tfrac{14}{3}p\cdot\gamma-4m-\tfrac{8}{3}p\cdot n\gamma\cdot n/n^2)(p^2-m^2)+p\cdot\gamma[6p^2+12(p\cdot n)^2/n^2]\\ &+m[-2p^2+8(p\cdot n)^2/n^2]-4p\cdot n\gamma\cdot np^2/n^2\\ &+(4w+2)\{8(\gamma\cdot p-3m)(p^2-m^2)\\ &+2p\cdot\gamma[-3p^2+14(p\cdot n)^2/n^2]+18m[p^2-4(p\cdot n)^2/n^2]\\ &-4p\cdot n\gamma\cdot np^2/n^2\}\\ &+(4w+2)^2(\gamma\cdot p-3m)(-\tfrac{1}{3}\gamma\cdot p+\tfrac{16}{3}p\cdot n\gamma\cdot n/n^2-2m)(\gamma\cdot p-3m)]\!]\end{aligned} \tag{9}$$

and the vacuum polarisation divergence is

$$\Pi_{\alpha\beta}(p)|_\infty = (-\eta_{\alpha\beta}p^2+p_\alpha p_\beta)K^2L[-2(p\cdot n)^2/n^2] + n^{-2}[(p\cdot n)^2\eta_{\alpha\beta}-(n_\alpha p_\beta+\eta_\beta p_\alpha)p\cdot n+p^2n_\alpha n_\beta]K^2L(-\tfrac{4}{3}p^2). \tag{10}$$

All these infinities are pervaded by axis dependences, and although the choices $\xi=-\frac{5}{12}$, $w=0$, $-\frac{1}{2}$ or $-\frac{9}{16}$ will accidentally remove *some* of them to one-loop order, there is nothing privileged about these values, nor will this do any good with photons or vector gauge mesons.

These n-effects, of course, arise in the self-gravitational quantum corrections, so it is not very surprising to find them in the matter field corrections. They remove any lingering hopes that the axial gravitational gauge is endowed with special virtue, and they prevent the establishment of any equivalences (Beven and Delbourgo 1980) between this gauge and Lorentz covariant gauge-fixing choices. Only if one is dealing with theories free of unrenormalisable infinities will there be any incentive to exhume the remains of the axial gravity gauge.

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† The principal value prescription for $k\cdot n$ poles in (4*b*) ensures that the absorptive parts of the amplitudes above have the correct two propagating gravitational degrees of freedom and that on-shell conservation laws are respected.